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Introduction

Performance in cycling is the result of the physiological capacity of the athlete and the chosen bike/material and their interaction.
Prior experiments have been done to establish several physiological parameters for a test subject. These include the Functional Threshold Power (FTP) and Critical Power (CP) in Watts which give is an indication of how much power the subject can produce aerobically. Also, the anaerobic capacity W' in kJ has been examined. Furthermore, the interaction between the cyclist and the bike has been studied by determining the aerodynamic drag for different cycling positions at different velocities as well as different bike setups.
Combining the propulsive and resistive forces acting on the bicycle, a prediction can be made about the cyclist's performance during a time trial. This research aims to validate an estimation of the time needed to finish a 400m time trial made by a mathematical model based on the cyclist's physiological capacity and the resistive forces. The prediction will be made for three different bike setups. First, with the rider's hands in the hoods, secondly with using aero extensions to reduce the frontal area and third with aero extensions and with a self-made disc wheel. The prediction of the hoods condition will be compared to a field trial with the same setup. This trial is used for the validation of the model based prediction.

Condition I: Hoods position

Condition II: Aero position

Condition III: Aero + self-made disc wheel from cardboard

Methods

Mathematical Model

For the prediction we use a simplification of the model described by Martin and colleagues (1998). The model to predict the time to finish a 400m time trial can be described by the following power equation:

$$(1)\hspace{1cm} P_{total} = P_{roll} + P_{kin} + P_{drag}$$

where the total power P_{total} will be measured with a power meter and is the sum of the power necessary to overcome the rolling resistance P_{roll}, the power necessary for changes in the kinetic energy P_{kin} and the power necessary to overcome aerodynamic drag forces P_{drag}. Here, P_{troll} is described by equation 2.

$$(2)\hspace{1cm} P_{roll} = m * \mu * g * v$$

where m is the mass of the rider and the bike (m = 73kg), μ is the rolling friction coefficient ($\mu = 0.005988$), g is the gravitational acceleration ($g = 9.81 \frac{m}{s^2}$) and v the velocity which shall be predicted and will be measured in the validation time trial using a GPS device. For this model, we are assuming a constant wind velocity of 2 $\frac{m}{s}$ which is the wind velocity from the day of the validation trial. This velocity is used by the website mywindsock.com, which delivers wind information for every road on the map.
P_{kin} can be described by equation 3.

$$(3)\hspace{1cm} P_{kin} = m * v * a$$

where a is the acceleration of the rider and the bike which as well will be changing over time in the prediction. P_{drag} can be described by equation 4.

$$(4)\hspace{1cm} P_{drag} = 0.5 * v^3 * \rho * C_d * A_P$$

where ρ is the air pressure which was gathered via the current weather information ($\rho = 1.1225 \frac{kg}{m^3}$), C_d is the drag coefficient and A_P is the projected frontal area. A_P for a certain setup on the bike can be determined by taking a photo of the rider and his bike from the front and counting the number of pixels that represent them. From a calibration object with known measures, the size of a single-pixel can then be determined and therefore also the size of the frontal area. The values for the hoods setup was $A_P = 0.425m^2$ and for the aero as well as the disc setup was $A_P = 0.314m^2$. The drag coefficient C_d and the rolling friction coefficient can be determined using a linear regression analysis approach explained by Debraux et al. (2011), where C_d can be calculated using the slope a of the linear regression equation.

$$(5)\hspace{1cm} C_d = \frac{a}{0.5*\rho*A_P}$$

here we came to $C_d = 0.845$ in the hoods position, $C_d = 1.001$ in the aero position and $C_d = 0.844$ in the aero position plus the disc wheel. μcan then be determined by using the intercept b of the regression

$$(6)\hspace{1cm} \mu = \frac{b}{m*g}$$

where we found an average value of $\mu = 0.005988$ which can be assumed constant. Now all parameters for the cycling model are given, which can be described in its entirety with equation 7.

$$(7)\hspace{1cm} P_{total} = m * \mu * g * v + m * v * a + 0.5 * v^3 * \rho * C_d * A_P$$

The minimum time our athlete needs for a given distance is determined by the maximum average speed v. This depends on how much power the athlete can push on average (P_{total}). To examine this, the physiological capacities have to be taken into account.

Physiological capacity

The physiological capacity is described by the aerobic and anaerobic system. This system is determined with an FTP test and a 3-min All-out CP test.

The FTP test estimates the aerobic capacity of the athlete. The athlete performs a 20-min best effort test which reflects 95% of his 60 min effort (FTP). Based on this test the $VO_{2}max$ is calculated with the following regression equation (Denham et al., 2017):

(8) $VO_{2}max = age * -0.313 + FTP [W*kg^{-1}] * 11.733+27.056$

Here, we found $VO_{2}max = 73.6$ $[ml*kg^1*min^{-1}]$

A 3-min Critical Power Test was used to estimate the anaerobic capacity W’ as described by Vanhatalo and colleagues (2007). W’ is the amount of energy available to the athlete anaerobically. To be more specific, this 3-min test is comparable to an extended Wingate-Test. The athlete has to start with pedaling as hard as possible and try to keep the power up. First the anaerobic capacity is emptied so that the average power of the last 30 seconds of this test represent the CP. The Energy W’ , which is used above the CP, is provided by the anaerobic system. To calculate this amount of energy, the integral of the whole power curve of the test was subtracted by the integral of the critical power over the three minutes. Here, we found W' = 22.5 kJ with CP = 301 W.
The maximal power output of the aerobic and aerobic system is shown in equation 9.

(9) $P(t) = \frac{W'}{t}+CP$

Prediction
The time needed to finish a 400m time trial shall be predicted by using the mathematical model described above. To facilitate the prediction and so that it can also be validated later by an experiment, we took the following assumptions:

- flat, smooth road
- constant headwind at time of event of $v_{wind}=2 \frac m s$ (gathered via mywindsock.com)
- anaerobic capacity cannot be fully emptied in 400 m
- Initial velocity of $v_{0}=0 \frac m s$
- Power outputs from 3 min test can be repeated

The start is from a standing start, so that the rider and the bike have to be accelerated to the maximum at first. To calculate the velocities reached by the athlete, we have chosen a numerical approach with time steps of 1 second. The produced power for each time step was taken from the 3 min all-out test to calculate the velocities at each time step. In the prediction, we assumed this power output could be repeated and at every time step would equal the sum power to overcome rolling friction, aerodynamic drag and changes in kinetic energy.

$$(10)\hspace{1cm} P_{total}(t) = m * \mu * g * v(t) + m * v(t) * a(t) + 0.5 * v(t)^3 * \rho * C_d * A_P$$

In a discrete-time model, the acceleration can be written as $a(t+1) = \frac{ v(t+1)-v(t)} t$. Now, a change in velocity can be written as a function of the initial velocity and the current power output possible. All calculations were performed using MATLAB [The MathWorks, Inc]. The full script used for that can be found in the appendix.

Results

We estimated the finish time for the time trial for the hoods position, aero bars position and aero bars position with the disc wheel. The validation test has only been done for the hoods position and the estimations for all three setups can be seen below in Figure 1.

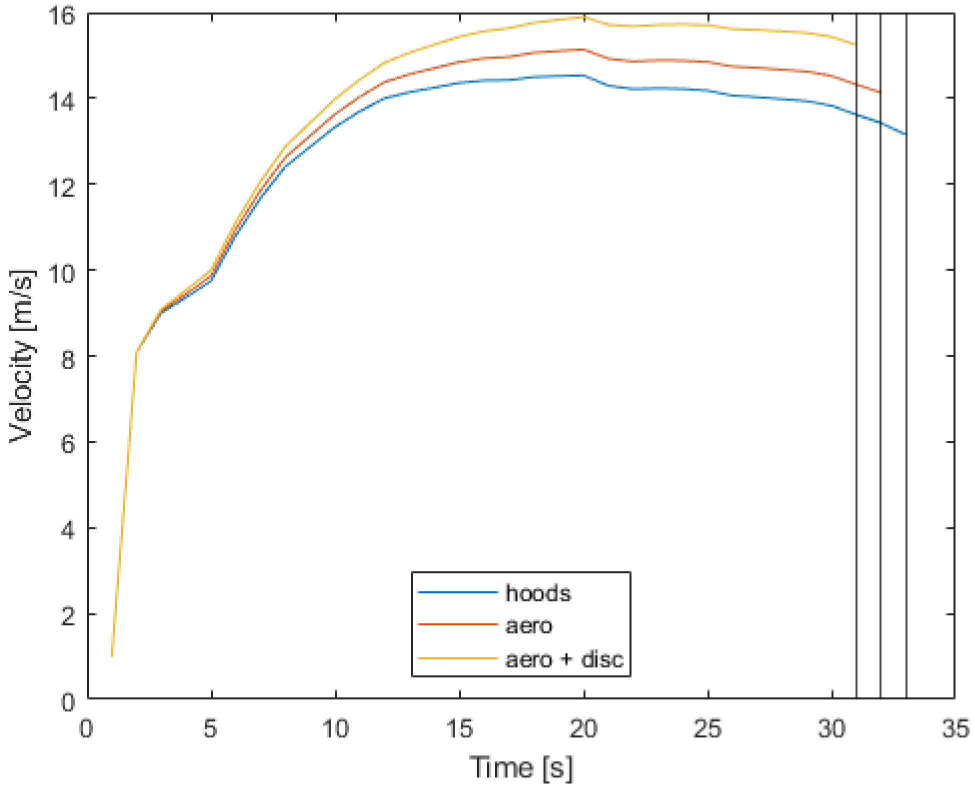

Figure 1: Predicted velocity curves over time for the three test conditions

Figure 2 shows the predicted velocities for the hoods condition in comparison to the velocities measured in the validation trial over time until the finish line was reached. The estimated finish time for the 400 m time trial was t_{end} = 33 s. The measured finish time for the 400 m validation time trial was t_{end} = 32 s. This corresponds to a prediction error of \delta = 3\%.

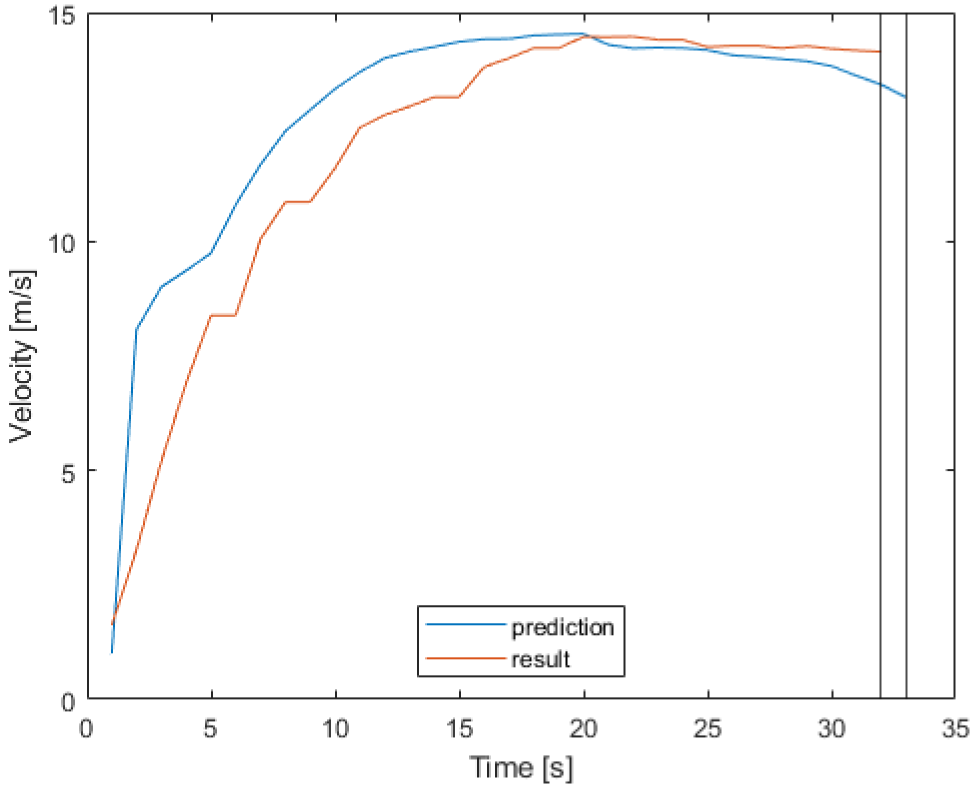

Figure 2: Predicted velocity curve over time vs measured velocity curve over time of the validation trial

Also, the measured power curve differed a bit from the predicted values as to be seen in figure 3. In the prediction we assumed, the power curve from the 3 min all-out test can be reproduced. In reality, the shifting of gears lead to drops within this curve.

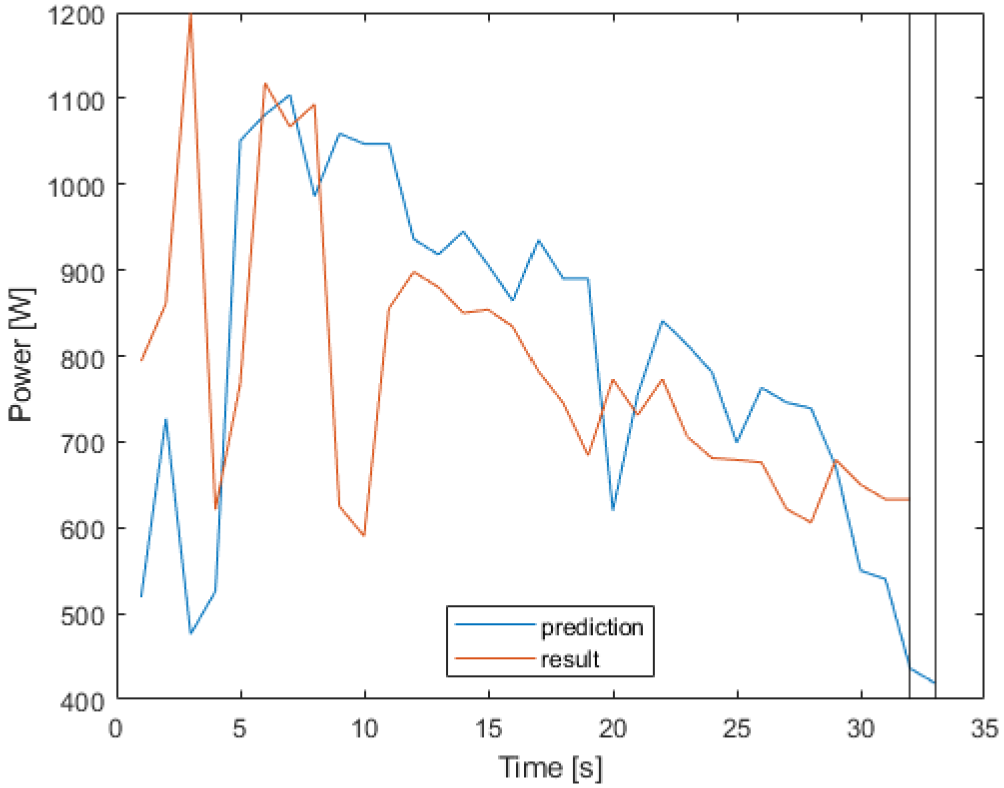

Figure 3: Predicted power curve vs power velocity curve over time

Discussion

The aim of this study was to design a mathematical model to predict the time our athlete would need for a 400m time trial and then validate this by letting the subject perform a real time trial. The time predicted by the model only varied by 1 second compared to the real time which corresponds to a prediction error of \delta = 3\% and therefore definitely lies within a reasonable range.

This difference can be explained by the structure of the study itself. On the one hand we have a theoretical mathematical construct and on the other hand a real world scenario with variables that cannot necessarily be predicted. Among other things these variables are:

- the rider had to shift gears at certain time steps which was not taken into account in the power curve of the prediction

- the wind was not necessarily constant during the test
- the asphalt was different from that used to determine rolling resistance
- the athlete's daily form may have been different from that used to determine the physiological capacity.
- wheel/crank bearing and drive train efficiency was not taken into account

Of course we could try to find approximations for these variables and integrate them into the model, but this would significantly increase the complexity of the model and the result would probably not be more accurate than the one we obtained. Moreover, a more complex model would also be accompanied by a worse generalization of the predictions.

Conclusion

The proposed mathematical model can validly predict time trial times for short efforts where the anaerobic energy cannot be emptied. Also, the model is limited to be used for straight, flat roads with constant environmental conditions (wind, rolling friction). For more complex courses, the model needs to be adjusted.

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